> There's a lady who's sure
> All that glitters is gold
> And she's buying a stairway to heaven

We open the song by giving some context. The lady, of course, is mathematics (or rather, the mathematical community), and her belief that all that glitters is gold corresponds to the belief of the 17th and 18th centuries that results with elegant proofs *had* to be true, regardless of rigor, regardless of ambiguity.

Counterexamples were not recognized as such, but rejected as pathologies or anomolies not fit to be called by the ideas they purported to instantiate.

However, a revolution was afoot, and it was widely recognized that these counterexamples could not be swept under the rug. It had seemed inevitable that any attempt at careful definition would lead to infinite regress, or something worse, and that these counterexamples could not be eradicated. But with the invention of non-Euclidean geometry and set theory, axiomatic systems took on a new life, and it seemed these fears were unfounded...

> When she gets there she knows
> If the stores are all closed
> With a word she can get what she came for

Here the stores being closed represents the failure of heuristic reasoning to completely identify and avoid these pathologies, and to find (and know it had found!) Truth.

But "with a word she can get what she came for" — perhaps with these new axiomatic systems, reasoning itself can be reduced to rock-solid syntatic rules, with no room for error!

> Ooooh, oooh, and she's buying a stairway to heaven

> There's a sign on the wall but she wants to be sure
> 'Cause you know sometimes words have two meanings

As we will see, "she's buying a stairway to heaven" is a sad description of this endeavor and its futility.

The comment about words having two meanings is a direct reference to the Gödel numbering used in his proof to assign metamathematical interpretations to number-theoretical results.

> In a dream by the brook, there's a songbird who sings
> Sometimes all of our thoughts are misleading

> Oooh, it makes me wonder
> Oooh, it makes me wonder

Back to history. The Russel-Whitehead program is well underway, and given recent successes of the new axiomatic reasoning, it seems hopeful that they will succeed. Still, there are lingering doubts...

For now, "it makes me wonder" is a vague uneasy. It will become more solid as the song progresses, but I will not draw attention to it.

> There's a feeling I get when I look to the west
> And my spirit is crying for leaving

"Looking to the west" references Aristotelean logic. Maybe the program has gone wrong on a horribly fundamental level.

> In my thoughts I have seen
> brings a smoke through the trees
> and the voices of those who stand looking

. . . and maybe we knew this, and forgot. The "voices of those who stand looking" are those of the Sophists, condemned the backwaters of history by Aristotle.

> Oooh, it makes me wonder
> Oooh, really makes me wonder
>
> And it's whispered that soon if we all call the tune
> Then the piper will lead us to reason
> And a new day will dawn for those who stand long
> and the forest would echo with laughter

These lines introduce the Russell-Whitehead program, and paint a seductive picture of the world should they succeed. The tune of course is *Principia Mathematica*, and the piper its authors.

It is important to note here that Gödel/Page seeks an axiomatization of mathematics just as much as any of us. He sees its beauty, and recognizes the tragedy in his destruction of it.

> Ohh
> If there's a bustle in your headroom[1], don't be alone there
> It's just an inkling of a daydream

Here we warn of the dangers of heuristic reasoning. The phrase "bustle in your headroom" refers to the creation of a mathematical theorem out of seemingly nowhere. He warns not to be alone when it happens for two reasons: first, because the pursuit of truth is also a pursuit of madness.

Secondly, as the Gödel result will imply, the space of true statements is far larger than any human can conceive of. If you wander out there alone, you may well find yourself completely lost, and then you'll be alone forever.

> Yes there are two paths - you can go back, but in the long run
> There's still time to change the road you're on
> And it makes me wonder
> Oh, oh,

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[1] At this point we are seriously deviating from the "official" lyrics. But in our defense, Robert Plant is well-documented as not knowing what the fuck he's saying ever.

This introduces the spectre of undecidable propositions. "In the long run there's still time..." means that no matter how many axioms you add, no matter how many undecidables you decide, there will still be more, and you'll be no less lost than you started.

The line "it makes me wonder" is finding its meaning.

> Your head is humming and it won't go, in case you don't know
> The piper's calling you to join him

As we see all these threats formed at human reasoning, the program to put it all on solid foundation takes an urgency; its seduction becomes desperation. Lord, how we want to join the piper.

> Dear lady, can you hear the wind blow, and did you know
> your stairway lies on the whispering wind

This is the Gödel result.

> (guitar riff)

This is the Gödel construction.

> And as we wind on down the road
> Our shadows taller than our soul
> There walks a lady we all know
> Who shines white light and wants it shown
> How everything still turns to gold

The lady, to reiterate, is mathematics. The phrase "shadows taller than our soul" is the statement that truth is stronger than provability — so strong, in fact, that there are truths which are logically inaccessible to us except as shadows: uncomputable numbers, undecidable theorems, nonmeasurable sets.

But nonetheless, there is beauty here. We shall not desert it.

> And if you listen very hard
> The truth will wander, you are lost!
> When all is one and one is all
> To be a rock and not to roll.

To "listen very hard" is to formulate a theory very carefully. Then "the truth will wander" refers to its ultimate inability to decide what it describes.

> And she's buying a stairway to heaven.

References

[1] Led Zeppelin (1971), *Stairway to Heaven*.

[2] Kurt Gödel (1931). *Uber Formal Unentscheidbare Satze der Principia Mathematica and Verwandter Systeme, I.*